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THE EVALUATION OF MUTUAL SUBSTITUTION ELASTICITY OF CAPITAL AND LABOUR FACTORS BY APPLICATION OF CES FUNCTION FOR ECONOMY OF AZERBAIJAN

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Abstract

This study estimates parameters of CES production function in a Mathcad system using non-linear ordinary least squares method (Markvart method) based on statistical data of republics of Azerbaijan and Kazakhstan. Identified parameter estimates were comparatively analyzed to reveal a number of findings.

For both countries, capital-labour substitution elasticity (σ) turned out less than one, which indicates insufficiency of labour, namely qualified labour (skilled labour) in both economies.

Azerbaijan have experienced windfall revenues from exploitation of natural resources (particularly, crude oil) in recent years. These revenues induced greatly the imports of capital-intensive products of sophisticated technologies, in other words capital imports. Naturally, scarcity of adequate labour that could deploy increased capital (skilled labour-intensive capital) resulted in decline of reciprocal elasticity of capital-labour substitution. Thus, it can be concluded that utilization of oil revenues to accumulate qualified labour (i.e. development of education, science, etc., technical specializations) would increase reciprocal elasticity of capital-labour substitution. Hence, expenditures on building qualified labour, including spending on education and science are preferred areas of efficient use of oil revenues.

Keywords: Capital, Labour, Substitution elasticity, Production function, Markward method.

JEL Classification Codes: C01, C02

Introduction

As you may know some production functions require the certain researches during evaluation of their parameters. The one with the most common feature and reflecting the neaclassic theory is the production function of Constant Elasticity of Substitution (CES).

$$Y = A_0 \cdot (\delta K^{-\rho} + (1-\delta)L^{-\rho})^{-\frac{\nu}{\rho}}$$

Here, Y-Gross domestic product, , K-Capital, L-labour force.

If we consider the effect of neutral technical progress according to Hicks the CES production function will be written as following:

$$Y = A_0 \cdot e^{\lambda t}(\delta K^{-\rho} + (1-\delta)L^{-\rho})^{-\frac{\nu}{\rho}}$$

Here, e is irrational and called figure of Eyler figure: $e \approx 2.72$, t is indicating time.

The following parameters should be valued:

A_0 is scale ratio ($A_0 > 0$) and its value depends on what is the unit of the determinant. If determinant is homogenous or shown by percentage, them the ratio shows the intensity of the production and equal to something around 1. δ-is allocation ratio. ν is a degree of gomogenity. ($\nu > 0$, λ is a level of technical progress and reflects the time pattern, ρ is a parametr showing the elasticity of substitution. ($\rho \geq -1$),

$$\sigma = \frac{1}{1+\rho} \tag{2}$$

In CES production function as in Cobb-Douglas production function, K and L are constant dependants of substitution limit. (see: box 1).

Box 1: The rate of substitution limit.

The rate of substitution of labour factor with capital factor in conditions when production level is constant is equal to additional capital required for decrease of one unit of labour.

$$X \in I : f(K, L) = Y_0 , \tag{B1.1}$$

Here: Y_0 – the given level of output

(B1.1) if we differentiate the iso-quantum we will obtain the following quotation:

$$\frac{\partial\ f(X)}{\partial\,(K)}dK + \frac{\partial\ f(X)}{\partial\,(L)}dL = 0$$

Considering that $dX = (dK, dL)$ *difined in this way, we obtain the following quuatation:*

$$MY(X)dX = 0 \tag{B1.2}$$

If we expand the quotation (B1.2),

$$MY_K dK + MY_L dL = 0 \tag{B1.3}$$

we will obtain the following quotation:

$$\left.\frac{dK}{dL}\right|_{iso-quantum} = \gamma_{L,K} = -\frac{MY_L}{MY_K} = \frac{(\frac{\partial f}{\partial L})}{(\frac{\partial f}{\partial K})} \qquad (B1.4)$$

The limit of substitution of factor K with factor L ($\gamma_{K,L}$) *could be defined as follow:*

$$\gamma_{K,L} = \left.\frac{dL}{dK}\right|_{iso-quantum} = -\frac{MY_K}{MY_L} = \frac{(\frac{\partial f}{\partial K})}{(\frac{\partial f}{\partial L})} \qquad (B1.5)$$

From combination of (B1.4) and (B1.5) we obtain the following:

$$\gamma_{K,L} = \frac{1}{\gamma_{L,K}} \qquad (B1.6)$$

We can define the rate of substitution rate though the elasticity ratios as following:

$$\gamma_{L,K} = -\frac{MY_L}{MY_K} = \frac{(\frac{\partial f}{\partial L})}{(\frac{\partial f}{\partial K})} = \frac{\varepsilon_L K}{\varepsilon_K L} \qquad (B1.7)$$

Although both Cobb-Douglas and CES functions are based on neo-classic theory they are significant differences between them. Thus as we know the substitution elasticity is a possibility of one factor to be replaced by other one. For example, capital and labour force, in Cobb-Douglas function this substitution elasticity is equal to one. In CES function this ratio can take any value. It is clear that during the evaluation of parameters for Cobb-Douglas function it is determined in advance that capital and labour has unit elasticity. If it is true than the parameters identified for Cobb-Douglas function can be misleading. In order to eliminate this shortage we should first of all evaluate the parameters of CES function and calculate the substitution elasticity of the factors. If it is equal to one than the function is equal to the Cobb-Douglas production function.

We would like to note that σ-substitution elasticity in function $Y = F(K, L)$ is identified as shown in box 2.

Box 2: Substitution elasticity

The possibility for substitution of the factors with each other shows various combination of production factors in case when the production function is able to maintain the constant production level. For example, substitution of local change between capital and labour factors in cases when all other conditions are equal, several points in special area could be the elasticity factor between capital and labour. The substitution elasticity between capital and labour could be defined as follow:

$$\sigma_{KL} = -\frac{d\ ln(K/L)}{d\ ln(M\ Y_K\ )/MY_L\ )} \qquad (B2.1)$$

$$\frac{\partial F}{\partial K} dK + \frac{\partial F}{\partial L} dL = 0$$

If we write it in more extend form, it looks like

$MY_K dK + MY_L dL = 0$

Here, MY_K *and* MY_L *show the limit of substitution of the production (GDP) for capital and labour respectively.*

$$MY_K = \frac{\partial F}{\partial K}$$

$$MY_L = \frac{\partial F}{\partial L}$$

*We can see from (B2.1), **the ratio of substitution between capital and labour** is equal to correlation between their changes in percentage and percentage change of their limit products.*

*In other words, **the ratio of substitution between capital and labour** is equal to correlation between their changes in percentage and percentage change of their limit products.*

The substitution elasticity shows the expanses (capital and labour) for maintenance of the same level of output and called iso-quantum curve.

Now let show the vice-versa elasticity of the substitution:

$$\frac{1}{\sigma_{KL}} = -\frac{d(\ln(\frac{MY_K}{MY_L})}{d(\ln(\frac{K}{L})} = \frac{(\ln(-\frac{dL}{dK}\Big|_{iso-quantum})}{d(\ln(\frac{K}{L})} \qquad (B2.2)$$

In general the differences between CES function and other production functions is shown in the Table 1 below.

Table 1. Production functions and their parameters

Types of production functions	Production functions Y=F(K,L)	σ –substitution elasticity	ε-production elasticity	Parameters
1	2	3	4	5
Linear	$Y = a_1 \cdot K + a_2 \cdot L$	∞	1	a_1 and a_2 are limit products for capital (K) and labour (L) respectively .
Cobb-Douglas	$Y = A \cdot K^{b_1} \cdot L^{b_2}$	1	$b_1 + b_2$	A-scale factor, A>0, b_1 and b_2 are elasticity factors for capital (K) and labour (L)

				respectively. $b_1 \geq 0$, $b_2 \geq 0$.
"Expenses-Output" (Leontyev)	$Y = \min\left(\frac{K}{c_1}, \frac{L}{c_2}\right)$ or $K \geq c_1 \cdot Y$ $L \geq c_2 \cdot Y$	0	1, $\frac{K}{c_1} = \frac{L}{c_2}$	c_1 and c_2 are the quantities of capital (K) and labour (L) for production of one unit of the production. $c_1 \geq 0$, $c_2 \geq 0$.
The analysis tools for production activity	$Y = \sum_{r=1}^{n} d_r \cdot y_r$ $\sum_{r=1}^{n} d_{Kr} \cdot y_r \leq K$ $\sum_{r=1}^{n} d_{Lr} \cdot y_r \leq L$	0	1	n- quantity of production tools, y_r- the intensity level of the tool, *r*=1,2,...,n. d_r- the production level when intensity is equal to 1., r=1,2,...,n. d_{Kr} and d_{Lr}-r-the quantity of capital (K) and labour (L) expenses for unit intensity of the tool.
Constant elasticity of substitution (CES)	$Y = A_0(\delta K^{-\rho} + (1-\delta)L^{-\rho})^{-\frac{\theta}{\rho}}$	$\frac{1}{1+\rho}$	θ	A_0–scale ratio ($A_0 > 0$), δ-allocation ratio, θ- the decree of homogeneity ($\theta > 0$), ρ – substitution elasticity (σ) characteristic ratio ($\rho \geq -1$).

The constant elasticity of substitution (CES) function ($\sigma = \frac{1}{1+\rho}$), combines the first tree functions:

1) if $\rho \to$ -1, the CES functions turns into the linear function ($\sigma = \infty$);

2) if $\rho \to 0$, the CES functions turns into the Cobb-Douglas function (σ=1);

3) if $\to \infty$, the CES functions turns into the "expense-output" production function (σ=0).

The theoretical aspects of evaluation of parameters of CES function. Markward method.

The CES function even after logarithmation remains not-linear. Therefore in order to evaluate the parameters of CES function, we should apply the least square method.

In general non-linear the least square method is presented in following way:

Let's guess that the variable Y is non-linear function showing the dependence of the last on on variables $X_{1,} X_2 ,...., X_n$.

$$Y = F (X_{1,} X_2 ,...., X_n)$$

However the parameters $a_{1,} a_2 ,...., a_n$ of variable $X_{1,} X_2 ,...., X_n$ *respectively are unknown.* Here, a_i – is parameter showing how variable X_i can affect the variable *Y.* The valuation of this parameter is required. For this purpose we have performed m times observation. As a result of observations we have identified respective variable $(X_{i1,} X_{i2} ,...., X_{in})$ $(i=1,2,...,m)$ for each variable Y_i.

In other words,

$$Y_i = F_i (a_{1,} a_2 ,...., a_n; X_{i1,} X_{i2} ,...., X_{in})+ U_i, \quad i=\overline{1,m}, \qquad (3)$$

Here, U_i –is deviation. In (3) $a_{1,} a_2 ,...., a_n$ we should find such parameters, that the values obtained during the observation were maximum close to these obtain in the theory. In other words, deviation U_i should be at lowest point. The parameters $a_{1,}$

a_2 ,...., a_n which meet this requirement are found by the least square method. In other words, the function below is being minimized:

$$S(a_1, a_2, \dots, a_n) = U_1^2 + U_2^2 + \dots + U_n^2 = \sum_{i=1}^{n} U_i^2 \rightarrow min \tag{4}$$

Due to the fact that the goal function S is non-linear in respect of parameters a_1, a_2 ,...., a_n the identification of its minimum faces certain problems during the application of Ferma theory. Thus the special derivative of function S in respect of parameters a_1, a_2 ,...., a_n and by equation of this derivative to zero finding the solution of quotation system faces number of the problems and even is not possible. Therefore the minimization method resolves this problem. We can relate the Newton – Gauss, Markward, Pauelov and Highbred methods to the method for minimization of S function.

Let's introduce the following vectors:

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_m \end{bmatrix}, F = \begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ \cdot \\ F_m \end{bmatrix}, U = \begin{bmatrix} U_1 \\ U_2 \\ \cdot \\ \cdot \\ \cdot \\ U_m \end{bmatrix}, a = \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{bmatrix} \tag{5}$$

Now we can show the problem in the following way:

We should find such point a*, which satisfy the following condition: if $U=Y-F$ function $S = U^T U$ should take the minimal value.

Here, vector U^T - is a transposed vector U.

The approximate value a^k obtained on k-th step of the iterative process the next approximate value a^{k+1} are linked with each other with correcting vector Δa.

$$a^{k+1} = a^k + \Delta a \tag{6}$$

The correcting vector Δa could be found in conditions of minimization using the following formula.

$$(A^T A)\Delta a = (-A^T U) \tag{7}$$

Here, $\Delta a = -(A^T A)^{-1} A^T U)$. The matrix A is the matrix of the first special derivative of U. In other words it is Yacobi matrix.

$$A = \begin{bmatrix} \frac{\partial U_1}{\partial a_1}, \frac{\partial U_1}{\partial a_2}, \ldots, \frac{\partial U_1}{\partial a_n} \\ \frac{\partial U_2}{\partial a_1}, \frac{\partial U_2}{\partial a_2}, \ldots, \frac{\partial U_2}{\partial a_n} \\ \ldots \\ \frac{\partial U_i}{\partial a_1}, \frac{\partial U_i}{\partial a_2}, \ldots, \frac{\partial U_i}{\partial a_n} \\ \ldots \\ \frac{\partial U_m}{\partial a_1}, \frac{\partial U_m}{\partial a_2}, \ldots, \frac{\partial U_m}{\partial a_n} \end{bmatrix}$$

The elements of matrix A are calculated for point $a = a^k$.

This formula is the basic formula in iteration in respect of Newton-Gauss formula. When we use the Newton-Gauss method the more non-linear characteristics of f(x) and deviation between initial value a^0 and minimum value, the more is the possibility of failure of iterative process.

Historically solution for minimization of the function with number of variables is made based on declining method of gradient. The meaning of this method is selection of direction of the vector.

$$d = -\nabla S$$

Here,

$$\nabla S = \frac{\partial S}{\partial a} = 2A^T U$$

However the direction selected under the gradient method could be far from the optimal level. In these cases Δa frequenty gives bad combinations. It was suggested to obtain the accurate vector Δa during the Newton-Gauss procedure by adding non-negative sign to the diagonal matrix of Liebenberg and Marward (7).

$$(A^T A + \nu^2 I)\Delta a = -A^T U \quad (8)$$

Here, the parameter I-nxn is single matrix. ν- is any quantity (it maybe equal to zero) and it is called Marward number.

This method eliminates the shortages of declining gradient and Newton-Gauss methods.

Thus,

$$\Delta a = -(A^T A + \nu^2 I)^{-1} A^T U$$

If $\nu = 0$ we came to Newton-Gauss method, in cases when ν is at higher value we obtain declining method of gradient:

$$\Delta a \approx -(\nu^2)^{-1} A^T U$$

Thus we can say that this method combined the declining gradient and Newton-Gauss methods. The main idea of this method is that in cases with high level of non-linearity (till the moment when distribution between iterative and searched solutions is large enough) we use the large quantities for ν (gradient method), when we approach to seached solution we the value of ν is decreasing. Thus process gives an opportunity to approach to the desired solution at shortest time.

Described above method is called Marward method. The following algorithm is proposed for identification of Marward number:

1) Initial value $\nu = \sqrt{0{,}001}$, $(\nu^2 = 0{,}001)$ is accepted;

2) When S –starts to decrease in one step figure ν increases $\sqrt{10}$ times $(\nu^2$ increases by 10 times);

3) The value of ν is increased by $\sqrt{0{,}1}$ times $(\nu^2$ by 0,1 times), obtained amount is accepted as new initial value.

1)-3) are the main iterations of this method (increase or decrease of S depends on level of non-linearity).

In order to reduce the volume of calculations and interpretations we suggest to use the following formula:

$$(\widetilde{A^T} A^T + \nu^2 I)\Delta a = -\widetilde{A^T}\widetilde{U} \tag{9}$$

Here,

$$\tilde{A} = \begin{bmatrix} A \\ \nu I \end{bmatrix}, \tilde{U} = \begin{bmatrix} U \\ 0 \end{bmatrix}.$$

Please note that the difference between the system equations bring about two conditions:

a) Expanded Yacobi $\tilde{A}$ matrix is obtained through addition of νI matrix with dimensions of nxn to the lowest raw of Yakobi matrix A;

b) $\tilde{U}$ is a vector of expanded reminders, it is obtained through the addition of zero components with the length of n to U- initial reminders.

3. The practice of evaluation of CES production function

The CES production function evaluated the parameters based on various methods on examples of various countries. For example, Mishra, SK (2006) valuated the CES production function based on non-linear regression using five optimization methods: Hooke-Jeeves Pattern Moves (HJPM), the Hooke-Jeeves-Quasi-Newton (HJQN), the Rosenbrock-Quasi-Newton (RQN), the Differential Evolution (DE) and the Repulsive Particle Swarm methods (RPS)). For the Eurozone Matthieu Lemoine, Gian Luigi Mazzi, Paola Monperrus-Veroni, Frédéric Reynes (2009) estimated the CES SEC function.

Indicated model for valuation of SEC production function was used in the economic models applied in former USSR. Such valuations were made by

M.Weitzman, N.Barkalov, A.Granberg, U.İsterli, S.Fisher and others for the period 1950-1987.

For example, M.Witzman took as initial data the indexes for production and capital for the perio 1950-1969 ($i = 1, 2, \ldots, 20, m = 20$) and evaluated the parameters of the CES function:

$$Y = 0,8044156 e^{0,02046463t} (0,6397585\ K^{-1,5} + 0,3602415\ L^{-1,5})^{-0,66}$$

R2 = 0,9994216 ; DW = 0,8125811

The substitution index is $\sigma = \frac{1}{1+\rho}$ = 0,4031043.

Based on results obtained M.Weitzman came to conclusion that the development of USSR's economy during the post-war period was due to the fact that the substitution elasticity was far above one. It is explained by the shortage of labour force during this period. At the same time M.Weitzman criticized Bogson and other for application of Cobb-Douglass function with substitution elasticity equal to one.

The other contribution of M.Weitzman was that unlike R.Solow and others, during alternative valuation of CES he was the first who applied the non-linear version of least square method. On other hand R.Solow tried to use the identity of theoretical determinants and observation results in evaluation of CES production function.

A.Granberg performed the following valuation of CES function for the period 1960-1985.

$$Y = 0{,}966(0{,}4074K^{-3{,}03} + 0{,}5926L^{-3{,}03})^{-\frac{1}{3{,}03}} e^{0{,}0252t}$$

R2 = 0,9982; DW = 1,76

R2 – is determination ratio and *DW* is dependent identified by the statistical method.

The substitution elasticity is equal to $\sigma = \frac{1}{1+\rho}$ =0,25. The value to the substitution elasticity obtained by the other scientists during the various periods were below one: 0,4 (M.Weitzman), 0,37-0,43 for various data row (U.Easterly və S.Fisher). In general, for USSR economy the substitution elasticity is equal to 0,4. This means that the substitution level between the factors is lower than that obtained under the Cobb-Douglass method (in Cobb-Douglass function the substitution elasticity in the production function is equal to one). Thus application of Cobb-Douglass production function for this function is not able to reflect the true picture.

4. Realization of the model and analysis of results

In order to evaluate the parameter of CES function following program was prepared:.

1) In order to evaluate the CES function based on Marward method it was represented in following way after logariphmation:

$$F_i = lnY_i = lnA_0 + \lambda(i-1) + \frac{\sigma}{\sigma-1}\ln\left(\delta K_i^{-\rho} + (1-\delta)L_i^{-\rho}\right) + U_i, \quad i = \overline{1,m}; \rho = \frac{1-\sigma}{\sigma} \quad (10)$$

2) After this the approximated equations of CES function are evaluated based on Kmeta method:

$$\tilde{F}_i = lnY_i = lnA_0 + \lambda(i-1) + (1-\delta)lnK_i + \delta\, lnL_i - \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\delta(1-\delta)(lnK_i - lnL_i)^2 + U_i, i = \overline{1,m}; \rho = \frac{1-\sigma}{\sigma} \quad (11)$$

During the independence period of Azerbaijan Republic the impact of capital and human resources on whole economy and particularly to the oil and gas sector should be evaluated. After this the valuation and comparative analysis of the CES function in Republic of Kazakhstan were implemented.

Initially the function (10) is evaluated based on Marward method. In this case the following figures are taken as initial approximation:

$A_0 = 1$;

λ=0,05;

δ=0,5;

ρ=0,5.

Nate that $\theta =$.

Based in data for the period 1990-1996 the CES function for Azerbaijan economy was evaluated based on Marward method and Kmeta approximation (11). Obtained results were approximately the same:

$$Y = 1,05733e^{-0,1919t}(0,3217946\ K^{0,666} + 0,6782054L^{0,666})^{1,4999}$$

R2 = 0,87346; DW = 2,394745

Substitution elasticity $\sigma = 3$.

Statistical characteristics show that model is adequate.

We also obtained the substitution elasticity for Azerbaijan much higher than one. As a result evaluation of economy of Azerbaijan is not complete if the Cobb-Douglass function is applied. In reality, during the evaluation Azerbaijani economy using the Cobb-Douglas production models the obtained results do not provide adequate results as in CES function. In other words, making in advance the substitution elasticity equal to one and implementation of valuation of the parameters will not provide the correct results. However obtained a- constant λ - the growth rate of technical progress in CES function were approximately the same as in Cobb-Douglass function. It indicates that the efficiency of the economy and effect of technical progress in the country estimated in Cobb-Douglass were once more verified in CES production function.

The value of allocation parameters of capital and labour force obtained in M.Weitzman model were contradicting. Under the Weitzman approach the K and

L parameters were equal to 0,6397585 and 0,3602415 respectively. However our results were 0,3217946 and 0,6782054 respectively.

Thus, based on results under the Weitzman model we can conclude that during the period of 1990-2006 Azerbaijani economy experienced the excess of labour force.

Considering that substitution elasticity for Azerbaijani economy under the evaluation of CES production function is higher than 1 (σ=3), we can agree with such conclusion. U.İsterli and S.Fisher came to conclusion that the low level of substitution elasticity between labour and capital in USSR economy was the main reason for stagnation. Thus in cases of low elasticity the excess of the capital is not provide the same growth in production of goods. We also came to conclusion that lack of capital provision for the existing labour force in 1990-1996 brought about decrease in the production.

For the period of 1994-2000 the valuation of parameters of CES function for Azerbaijan in Mathcard Application Package shown that the substitution elasticity between labour and capital is close to one (σ=*1,000*3). Thus during this period the balance between the labour force and provided capital was re-established. In other words, during this period the qualification and education of the labour force was in accordance with provided capital.

Note that the evaluated results of the CES function for economies of Azerbaijan shows that that substitution elasticity between the factors (capital and labour) is less than one. This means that there is lack of experienced and qualified labour force for utilization production facilities.

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