FUZZY LINGUISTIC FORECASTING OF SOCIAL MOBILITY

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ABSTRACT:
In the article the mobility of social groups is estimated and predicted in order to investigate and analyze the wellbeing situation in the context of sustainable development in the Republic of Azerbaijan, to make proposals and recommendations for development in this direction. The examining of welfare, social inequality, and poverty has concluded that these indicators are not always sufficient to comprehend. In order to gain a complete picture, it is necessary to study income mobility or change of population income over time. If social mobility is high enough, concerns about the uneven distribution of income in the society can be diminished, resulting in unnecessity for redistribution of income at the societal level. In this paper we estimated mobility for social strata based on the H. Theil, and G. S. Fields and E. A. Ok indices and forecasted data through a fuzzy linguistic Markov chain for the next time period. Constructed models can be useful in the development of policies for poverty elimination and sustainable economic development.

Key words: social groups, social mobility, fuzzy linguistic markov chain

Jel classification: C02, J60

I. Introduction

Mobility is an important concept in several branches of social science and economics. The way it has been conceived has depended on the particular application or even the particular data set under consideration. Different parts of the literature have focused on income or wealth, wage, educational mobility, and mobility in terms of social class. As a consequence of this diversity, the measurement of mobility is an intellectual problem that has been addressed from many different standpoints. Mobility measures are sometimes defined, explicitly or implicitly, in relation to a specific dynamic model, sometimes as an abstract distributional concept similar to inequality, polarization, dispersion, and so on (Cowell and Flachaire, 2018).

The examining of welfar e, social inequality, and poverty has concluded that these indicators are not always sufficient to comprehend. In order to gain a complete picture, it is necessary to study income mobility or change of population income over time (Fields and Ok, 1996a). Van Kerm (2004) classifies income mobility as follows: growth, dispersion, and exchange mobility. The first of these comprise an increase in mean income of the distribution produced by economic growth. The dispersion component evaluates the degree to which income convergence occurs, studying the variation in the inequality of distribution without income being reranked. Lastly, the exchange component shows the magnitude of the rerankings among incomes.

In the article growth, dispersion, and exchange mobility are estimated in the time intervals of 2009–2017. In particular, mobility indices are estimated as proposed by H. Theil, and G. S. Fields and E. A. Ok which calculates mobility based on total income per capita, then the data are forecasted for the next period based on fuzzy linguistic Markov chain.

Section II of the article provides a theoretical framework for measuring social mobility; Section III presents the estimation of social mobility indices on social groups; Section IV presents social mobility forecasting.

II.Theoretical basis for assessment of social mobility

In order to highlight the advantage of the mobility index that will be applied, there is necessity to compare it with existing ones. Some mobility indices that are well constructed and widely accepted have been classified by M.Peng and et al (2010).

When justifying an index, there are some properties that must be taken into consideration to make comparisons: homogeneity (H), translation invariant (TI), decomposability (D), population consistency (PC), monotonicity (M), growth sensitivity (GS), and distance dimension (DD).

The comparison results are listed in Table 1, and we can see that:

(i) Each index obey variance axioms (properties), but all of them differ mainly in the distance functions.

(ii) No.7 and No.8 generalize the form of the distance function.

(iii) No.7 and No.8 are in accordance with each other, and their only difference lies in whether the distance function *f* is continuous or discrete.

There are a few points to further explain: both No.1 and No.2 are proposed by G. S. Fields et al., but only No.1 satisfies homogeneity (H) or translation invariant (TI). Moreover, No.2 formula is not a pure absolute income mobility index. Such an interesting phenomenon implicates that even the same person is in dilemma in how to characterize the income mobility due to its diversity. Secondly, No.5 is a general index when compared to No.2. When the function g has a determined form, $|y_i - x_i|$, No.5 satisfies homogeneity (H) and translation invariance (TI). Finally, all indices from No.1 to No.6 are equal-weighted, that is, every individual make an equal contribution to the total mobility. No.6 and No.7 have considered the effect of different weights

Table1. The comparison of absolute income mobility indices

№	Formula	H TI D PC M GS DD(p)	f_i	Papers
1	$\sum_{i=1}^{n} \lvert y_i - x_i \rvert$	√ √ √ √ √ √ 1	$\lvert y_i - x_i \rvert$	Fields and Ok (1996a), (1996b)
2	c($\frac{1}{n}\sum_{i=1}^{n} \lvert \log y_i - \log x_i \rvert$)	√ √ √ √ √ √ 1	$\lvert \log y_i - \log x_i \rvert$	Fields and Ok (1999), Fields(2006), (2007)
3	$c(\sum_{i=1}^{n} \lvert y_i - x_i \rvert^{p})^{\frac{1}{p}}$	√ √ √ √ √ √ p	$\lvert y_i - x_i \rvert$	Matra and Ok (1998)
4	$(\frac{1}{n}\sum_{i=1}^{n} (y_{i-} x_i)^2)^{\frac{1}{2}}$	√ √ √ √ √ √ 2	$(y_i - x_i)$	Dardanoni (1993), DAgostino and Dardanoni (2006),(2009a)
5	$(H(\frac{1}{n}\sum_{i=1}^{n} (g(y_i) - g(x_i))^2))^{\frac{1}{2}}$	√ √ √ √ √ √ 2	$(g(y_i) - g(x_i))$	DAgostino and Dardanoni (2009b), Checchi and Dardanoni (2002), (2006)
6	$\sum_{i=1}^{N} \frac{\int_{T}^{T'} \lvert x_i(t) - x_i(t - \Delta t) \rvert}{N(T' - T)}$	√ √ √ √ √ √ 1	$\lvert x_i(t) - x_i(t - \Delta t) \rvert$	Ding and Wang (2006)
7	$\iint d(x,y,F)dF(x,y)$	√ √ √ √ √ √ p	$d(x,y,F)$	Van Kerm (2004), (2006)
8	$(\sum_{i=1}^{n} f_i(d_i)^p)^{\frac{1}{p}}$	√ √ √ √ √ √ p	$f_i(d_i)$	Peng and et al (2010)

In addition, when the No.8 index adopts the equal weight, it is accordant with No.3. Furthermore, if the value of *p* is 1, then No.8 takes the form of No.1; if the value of p is 2, then No.8 takes the form of No.4. If we consider the welfare factor or ordinal ranks, the No.8 can take the form of No.2 and No.5. As for what is optimal *p,* it depends on the researcher's point of view about the income mobility since the mobility is multi-facet. In all, the index No.8 is similar to the formula proposed by Van Kerm (2006), and it is a generalized form for many other indices.

In this study, we use mobility estimation mothodology used by J.P.Rodriguez and et al. (2010) and apply it in a country level.

Let $X = (x_1, \dots, x_N)$ be the initial income distribution for N households. Then equivalent income vector X^e is defined through dividing money income by equivalence (e) scale. Thus, for household *i* the equivalent income is defined:

$$X_i^e = \frac{x_i}{e(N_i)} \quad (1)$$

where N_i is the number of household members, and e is the equivalence scale, where $1 \leq e \leq N_i$. It means that, the needs of a household grow with each additional member but – due to economies of scale in consumption – not in a proportional way. Needs for housing space, electricity, etc. will not be three times as high for a household with three members than for a single person. With the help of *equivalence scales* each household type in the population is assigned a value in proportion to its needs. The factors commonly taken into account to assign these values are the size of household and the age of its members (whether they are adults or children). A wide range of equivalence scales exist, but we consider only common methodology developed by OECD . In the 2008 and 2011 publications of the Organization for Economic Co-operation and Development, the comparison of income distribution and poverty comparisons across countries is calculated by dividing household income by the square root of the number of households. (http://www.oecd.org/eco/growth/OECD-Note-EquivalenceScales.pdf)

It implies that, four-person houshold has needs twice as large as one composed of a single person.

Thus, we consider the parametric scale proposed by Buhmann et al. (1988) and apply OECD 2011 methodology of "square root scale":

$$e(N_i) = N_i^{\alpha}, \; 1 \geq \alpha \geq 0, \quad (2)$$

where we accept α = 1/2.

Additionaly, each household can be weighed by the number of household members (Ebert, 1967) Which is developed and as a methodology generalized by OCED as "OECD equivalence scale" and "OECD – modified scale".

Now we can adjust formulas for equivalent incomes for social groups (see table 3) based on methodology mentioned above:

$$X_i = \frac{\bar{P}_i\, n}{\sqrt{n}} = P_i\sqrt{n} \quad (3)$$

Where, $\bar{P}_i$ is mean of per capita income intervals (see table 2), X_i is equivalent household income for income groups.

Then we can develop the mean of equivalent incomes formula for social groups:

$$\mu_x = \sum_{i=1}^{n} X_i\omega_i \quad (4)$$

Let to assume that, the vector of equivalent incomes X^e is ranked in ascending order:

$$0 \leq x_1^e \leq x_2^e \leq \cdots \leq x_N^e \quad (5)$$

Later, to estimate mobility index in the initial period the way proposed by H. Theil (1967) is applied:

$$T(X^e) = \frac{1}{N}\sum_1^N \frac{x_i^e}{\mu_X}\ln\frac{x_i^e}{\mu_X} \quad (6)$$

Then, we adjust this formula for social groups (described in section 2) by changing arithmetic average into weighted average:

$$T(X_\omega^e) = \sum_{i=1}^{n}\left(\frac{x_i^e}{\mu_x}\ln\frac{x_i^e}{\mu_x}\right)\omega_i \quad (7)$$

where $i = 1, \dots, n$ is the number of income intervals within a social group (n is conditional and changes for each social group), μ_x is the mean of equivalent incomes, and ω_i is weights of income groups according to table 2 in the initial period.

Distribution of equivalent incomes for the last period is: $Y^e = (y_1^e, y_2^e, \dots, y_N^e)$, where Y^e is ranked from lowest to highest. Thus, Theil inequality index adjusted for social groups in the final period is:

$$T(Y_\omega^e) = \sum_{i=1}^{n} \left(\frac{y_i^e}{\mu_Y} \ln \frac{y_i^e}{\mu_Y}\right) \omega_i \quad (8)$$

where μ_Y is the mean of equivalent incomes in the final period.
Mobility is measured using the approach proposed by G. S. Fields and E. A. Ok (1999), considering the transformation $X^e \rightarrow Y^e$

$$M\,(Y^e, X^e) = \frac{1}{N}\sum_{i=1}^{N} |\ln(y_i^e) - \ln(x_i^e)| \quad (9)$$

Using the same way mentioned above we adjust it for social groups:

$$M\,(Y_\omega^e, X_\omega^e) = \sum_{i=1}^{n}(|\ln(y_i^e) - \ln(x_i^e)|)\omega_i \quad (10)$$

Total mobility is decomposed into three elements: mobility due to growth (M^G), mobility resulting from dispersion (M^D) and exchange mobility (M^E). To this end, the following transformation function is considered according to Van Kerm (2004):

$$X^e \rightarrow \mu X^e \rightarrow Z^e \rightarrow Y^e \quad (11)$$

where $\mu = \frac{\mu_Y}{\mu_X}$ is the mean of of equivalent incomes in the intermediate period, Z^e $(z_1^e, z_2^e, \dots, z_N^e)$ – is intermediate distribution, defined as final distribution Y^e of equivalent income, ranked according to X^e.

Thus, the components of total mobility is obtained:

$$M^G(Y^e, X^e) = M(\mu X^e, X^e) - M(X^e, X^e) = \frac{1}{N}\sum_{i=1}^{N} |\ln(\mu x_i^e) - \ln(x_i^e)|$$

$$M^D(Z^e, Y^e) = M(Z^e, X^e) - M(\mu X^e, X^e) = \frac{1}{N}\sum_{i=1}^{N} |\ln(z_i^e) - \ln(x_i^e)| - \frac{1}{N}\sum_{i=1}^{N} |\ln(\mu x_i^e) - \ln(x_i^e)|$$

$$M^E(Z^e, Y^e) = M(Y^e, X^e) - M(Z^e, X^e) = \frac{1}{N}\sum_{i=1}^{N} |\ln(y_i^e) - \ln(x_i^e)| - \frac{1}{N}\sum_{i=1}^{N} |\ln(z_i^e) - \ln(x_i^e)| \quad (12)$$

Thus, mobility components adjusted for social groups take the form:

$$M^G\,(Y^e_\omega, X^e_\omega) = \sum_{i=1}^{n}(|\ln(\mu x^e_i) - \ln(x^e_i)|)\omega_i$$
$$M^D(Z^e_\omega, Y^e_\omega) = \sum_{i=1}^{n}(|\ln(z^e_i) - \ln(x^e_i)|)\omega_i - \sum_{i=1}^{n}(|\ln(\mu x^e_i) - \ln(x^e_i)|)\omega_i$$
$$M^E(Z^e_\omega, Y^e_\omega\,) = \sum_{i=1}^{n}(|\ln(y^e_i) - \ln(x^e_i)|)\omega_i - \sum_{i=1}^{n}(|\ln(z^e_i) - \ln(x^e_i)|)\omega_i \quad (13)$$

Furthermore, structural mobility M^S is identified by Van Kerm (2004) as the sum of the last two components: the dispersion and exchange mobility:

$$M^S(Y^e, X^e) = M^D(Y^e, X^e) + M^E(Y^e, X^e) \;\; (14)$$

Structural mobility excludes income mobility from the average growth of the regions.

Estimation social mobility indicators of social groups

According to the State Statistics Committee of Azerbaijan distribution of households by income and weights in the number of households are summarized in Table 2 by years.

Table 2. Household incomes and weights of each income groups

№	2009		2013		2017	
	Income	Weight	Income	Weight	Income	Weight
1	0-65 manat	0	0-95 manat	0.002	0-110	0
2	65.1-70.0	0	95.1-100.0	0.003	110,1-120,0	0.007
3	70.1-75.0	0.018	100.1-105.0	0.009	120,1-130,0	0.008
4	75.1-80.0	0.051	105.1-110.0	0.011	130,1-140,0	0.014
5	80.1-85.0	0.046	110.1-115.0	0.021	140,1-150,0	0.02
6	85.1-90.0	0.058	115.1-120.0	0.029	150,1-160,0	0.03
7	90.1-95.0	0.061	120.1-125.0	0.03	160,1-170,0	0.047
8	95.1-100.0	0.052	125.1-130.0	0.036	170,1-180,0	0.054
9	100.1-105.0	0.05	130.1-135.0	0.05	180,1-190,0	0.065
10	105.1-110.0	0.051	135.1-140.0	0.043	190,1-200,0	0.062
11	110.1-115.0	0.048	140.1-145.0	0.053	200,1-210,0	0.066
12	115.1-120.0	0.049	145.1-150.0	0.05	210,1-220,0	0.061
13	120.1-125.0	0.048	150.1-160.0	0.095	220,1-230,1	0.061
14	125.1-130.0	0.045	160.1-170.0	0.09	230,1-240,0	0.055
15	130.1-140.0	0.082	170.1-180.0	0.077	240,1-250,0	0.05
16	140.1-150.0	0.067	180.1-190.0	0.063	250,1-270,0	0.085
17	150.1-160.0	0.052	190.1-200.0	0.055	270,1-300,0	0.09
18	160.1-180.0	0.073	200.1-225.0	0.106	300,1-350,0	0.098
19	180.1-200.0	0.046	225.1-250.0	0.064	350,1-400,0	0.05
20	200.1-240.0	0.052	250.1-300.0	0.061	400,1-450,0	0.029

21	240.1-280.0	0.021	300- more	0.052	450,1-500,0	0.019
22	280 – more	0.03	-	-	500,1-550,0	0.011
23	-	-	-	-	550-more	0.018

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G. J. Imanov (2013) used the partitioning of households into the social strata based on income comparison with poverty line or minimum living subsistence. Different applications of the same method can be seen in the works of Russian scientists T. Bogomolova (2011) and V. Bobkov (2009). Using the data in Table 2, social stratification can be carried out as shown in Table 3 below.

Table 3. Social stratification by income

Social strata	2009		2013		2017	
	PL=84.0	Percent of population	PL=116.0	Percent of population	PL=155.0	Percent of population
1.Very poor- 0,5 PL	0-42.0	0	0-58.0	0.002	0-77.5	0
2. Poor 0,5-1,0 PL	42.0-84.0	0.115	58.0-116.0	0.044	77.5-155.0	0.079
3. Low satisfied 1,0-2,0 PL	84.0-168.0	0.663	116.0-232.0	0.777	155.0-310.0	0.696
4. Moderate satisfied 2,0-3,0 PL	168.0-252.0	0.171	232.0-348.0	0.125	310.0-465.0	0.177
5. High satisfied 3,0 PL	252.0-more	0.051	348.0-more	0.052	465.0-more	0.048

Using the data in Table 2 and Table 3, the social mobility indices - Theil index and index based on aggregate income per person proposed by Fields and Ok are estimated namely based on formulas (4), (6), (7). However, J.P. Rodriguez and et al. (2010) in the estimation of social mobility indices take the number of households as a constant, but in this problem the number of households covers the whole country, so in the estimation of the H. Theil, and G. S. Fields and E. A. Ok indices the total number of households is taken as a constant. That is, the social mobility indices are estimated based on the change in income and taking into account the number of people in social groups in the final periods.

Thus(, as an example,) the G. S. Fields and E. A. Ok index for high satisfied group is estimated as follows:

Firstly, we calculate mean of per capita income intervals, and average household income for income groups:

P_{23}^{2009} = 260.05 Y_{23}^{2009} = 551.65 P_{23}^{2017}= 475.05 Y_{23}^{2017} = 963.08

P_{24}^{2009} = 280.00 Y_{24}^{2009} = 593.97 P_{25}^{2017} = 550 Y_{25}^{2017} = 1115.02

P_{25}^{2017} = 550 Y_{25}^{2017} = 1115.02

μ_Y = 1043.29

Then, we calculate mobility indices:

$M_{FO}^{HS}\ (Y_\omega^e, X_\omega^e)$= $(\ln(963.08) - \ln(551.65)) \times 0.019 + ((\ln 1064.44) - \ln(593.97)) \times 0.011 + ((\ln(1115.02) - \ln(593.97)) \times 0.018$ = 0.03

$M_T^{HS}(Y_\omega^e, X_\omega^e)$= $(\frac{963.08}{1043.29} \ln \frac{963.08}{1043.29}) \times 0.019 + \left(\frac{1064.44}{1043.29} \ln \frac{1064.44}{1043.28}\right) \times 0.011 + (\frac{1115.02}{1043.29} \ln \frac{1115.02}{1043.29}) \times 0.018 = -0.0014 + 0.000225 + 0.001279 = 0.00010$

Where, M_{FO}^{HS} and M_T^{HS} denote Fields and Theil mobilty indices respectively.
All other results are shown in Table 4.

Table 4. Social mobility indices

Social strata	Theil index			Fields and Ok index
	2009	2013	2017	$M(Y^e, X^e)$
1.Very poor	0.00000	0.02000	0.00000	0
2. Poor	0.00012	0.00004	0.00034	0.04
3. Low satisfied	0.01018	0.01200	0.00952	0.38
4. Moderate satisfied	0.00103	0.00034	0.00097	0.11
5. High satisfied	0.00003	0.00000	0.00010	0.03

At the next stage, the mobility components: growth mobility (M^G), dispersion mobility (M^D), exchange mobility (M^E) and as a total mobility structural mobility (M^S) are estimated (Table 5).

Table 5. Social mobility and its components

Social strata	Components of mobility			Structural mobility
	M^G	M^D	M^E	M^S
1.Very poor	0	0.40	-0.40	0
2. Poor	0.06	1.38	1.58	2.96
3. Low satisfied	0.39	3.17	2.51	5.67
4. Moderate satisfied	0.10	0.15	1.60	1.75
5. High satisfied	0.03	0.14	1.60	1.74

The estimation results in all cases show that, mobility indices for Low satisfied group were high, which indicates very responsive mobility. Then comes indices for Moderate satisfied and Poor groups respectively. High satisfied group showed the least mobility when very poor group was completely immobile.

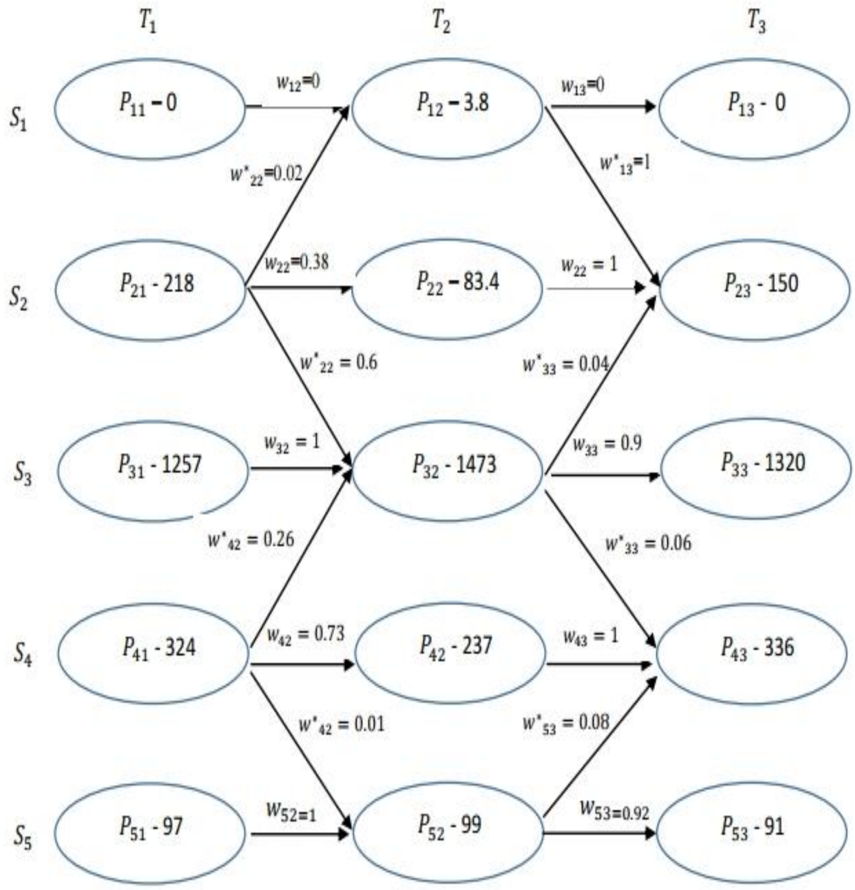
One possible explanation of the results is that the increase of social mobility is accompanied by changes in the social status of people, thus reducing inequality in society. That is, if a citizen earns less than the average monthly income in the economy today, he can earn more tomorrow. If social mobility is high enough, concerns about the uneven distribution of income in the society can be diminished, resulting in unnecessity for redistribution of income at the societal level. Reducing the need for redistribution of income can ultimately lead to income inequality.

Obviously only governments can undertake income redistribution programs. Some governments carry out this program with toughness, some do it in a relaxed pace. Addressing income inequality is not a real answer. Just the wealth gap must be addressed which is the cause of the income gap. If the wealth base kept unchanged any reduction in income gap will be ineffective. On top of that, governments' cash-transfer programs are usually charity programs. Charity programs are excellent as temporary relief, they cannot give permanent solution to the problem. Rather they hide problem. Central point of the proposal to redesign the economic framework is to move from personal interest driven economics to both personal and collective interest driven economics. There are some crucial ways to fight income inequality. First one is creating social business, defined as a non-dividend company to solve human problems such as, housing for the poor, health care, renewable energy, nutrition, water, nursing college, and many more. Then comes technological development. The combined power of the youth, technology, and social business can become an irresistible force. Education has the crucial role to bring this problem to the consciousness of people. Reorientation of education system is vital. Human beings has to be seen as go-getters, creator of new horizons, and problem solvers (Yunus, 2016). To get clear vision about economic condition in Azerbaijan some papers are useful from this point of view, see G. J. Imanov (2017) and Hasanli (2013).

III. Forecasting of social mobility indices.

The dynamics of households given in Table 3 along the social strata can be expressed in the form of a graph as shown in Figure 1.

Figure 1. Graph of social mobility dinamics.



In the graph S_i(i = 1,...,5) denotes economic strata of population , T_i(j = 1,...,3) – years of analysis, P_{ij}(i = 1,...,5;j = 1,…,3) – number of people in the economic strata in the corresponding year, w_{ij}^* and w_{ij} - portion of people moving to other strata and remaining in the same strata respectively.

It is obvious that, in all cases between social strata the number of ascending movements equals the number of descending movements.

Since the phenomenon of social mobility is consistent with the Markov chain process, this theory is used to forecast social mobility see Zadeh (1976), G. J. Imanov and Akperov (2013), Cheong and Hui (2001).

Consequently, G. S. Fields and E. A. Ok mobility indices, estimated for each socio-economic strata have to be fuzzified primarily see İmanov and et all (2003), Zadeh (1975). Assuming the mobility index is a linguistic variable, the range of obtained mobility data can be divided into three intervals: L(0-0.12), M(0.11-0.26), H(0.24-0.38).

Where, L – low, M – medium, H – high are linguistic terms. In other words, the mobility variables get their values within these intervals. We can assume the fuzzy interpretation of these linguistic variables according to membership functions given below:

μ_L = 1/0 + 0.7/0.03 + 0.5/0.04 + 0.2/0.07 + 0/0.10
μ_M = 0.1/0.11 + 0.4/0.14 + 1/0.19 + 0.6/0.22 + 0/0.25
μ_H = 0/0.26 + 0.2/0.28 + 0.4/0.30 + 0.7 /0.35 + 1/0.38
μ_{VL} = 1/0 + 0.49/0.03 + 0.25/0.04 + 0.04/0.07 + 0/0.10
μ_{VM} = 0.01/0.11 + 0.16/0.14 + 1/0.19 + 0.36/0.22 + 0/0.25
μ_{VH} = 0/0.26 + 0.04/0.28 + 0.16/0.30 + 0.49 /0.35 + 1/0.38

Where, added VL – very low, VM – very medium, VH – very high are concentrated forms of the above given terms.
The status vector S_{2017} = (0, 0.04, 0.38, 0.11, 0.03) of social mobility indices in the economic strata for the last period (2017) is in line with the linguistic social mobility vector (VL, L,VH, VM, L).

According to the analysis of Figure 1. the matrix of transition from one economic stratum to another can be represented as follows:

$$T_{2017} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \left\| \begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0.04 & 0.9 & 0.06 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.08 & 0.92 \end{array} \right\| \end{array}$$

In order to forecast social mobility indices for 2018 for each social strata the fuzzy linguistic Markov chain is applied:

$$S_{2018} = S_{2017} \circ T_{2018} = S_{2017} \circ T_{2017}^2 \quad (18)$$

$$T_{2017}^2 = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \left\| \begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0.04 & 0.9 & 0.06 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.08 & 0.92 \end{array} \right\| \end{array}$$

As it is known, by *maxmin* multiplication of transition matrices, the resulting transition matrix is the limit matrix itself. And the stationary solution of the fuzzy system does depend on the initial state, even though the system can freely move from one state to another. This is a crucial distinction between fuzzy Markov chains and probabilistic Markov chains. Markov chain is referred as the ergodic chain if it is aperiodic, convergent and transition matrix has identical rows (Avrachenkov and Sanchez, 2002).

But, in this case of interest the transition matrix is aperiodic and convergent, but not ergodic.

Assuming again that the probability of transition between social strata is a linguistic variable, the probability range [0,1] can be divided into three intervals: L(0-0.3), M(0.3-0.7), H(0.7-1).

If the elements of a discrete transition matrix in the formula (11) replaced with linguistic variables, the following fuzzy linguistic transition matrix can be obtained:

$$T_{2018} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \left\| \begin{array}{ccccc} VL & VH & VL & VL & VL \\ VL & VH & VL & VL & VL \\ VL & VL & H & VL & VL \\ VL & VL & VL & VH & VL \\ VL & VL & VL & VL & H \end{array} \right\| \end{array}$$

The elements of the vector in the expression (10) are defined by the following formula:

$$S_i = \bigcup_n (S_i \wedge T_i) \qquad (20)$$

By estimation of elements S_i forecasted fuzzy linguistic vector is obtained:

$$S_{2018} = (\text{VL, L, H, M, L})$$

The elements of this vector are determined in the following order:

S_1 = U[(VL∧VL), (VL∧VL), (VL∧VL), (VL∧VL), (VL∧VL)] = VL
S_2 = U[(L∧VH), (L∧VH), (L∧VL), (L∧VL), (L∧VL)] = L
S_3 = U[(VH∧VL), (VH∧VL), (VH∧H), (VH∧VL), (VH∧VL)] = H
S_4 = U[(M∧VL), (M∧VL), (M∧VL), (M∧VH), (M∧VL)] = M
S_5 = U[(L∧VL), (L∧VL), (L∧VL), (L∧VL), (L∧H)] = M (14)

The results show that there has been a subtle change in the social mobility indices for S_3 – group which denotes low satisfied people. Thus, the social mobility index is going to descend from very high to high. The inexistence of change in other social strata mobility indices is due to the fact that the transition matrix covers only recent data.

CONCLUSIONS

In the article, the mobility indices of social groups were calculated based on the methods proposed by Theil, Fields and Ok separately for 2009, 2013, and 2017. It was found that the mobility indices for the low satisfied group were high, medium for the moderate satisfied group, and low for other strata.

The forecast for next year carried out by fuzzy linguistic Markov chain suggest that there will be no significant change. As high mobility indexes indicate that there is no need for redistribution of income in society, research in this area can be useful in the development of social policy programs.

REFERENCES

L.A.Zadeh. The Concept of Linguistic Variable and its Application to Approximate Reasoning – II* //American Elsevier Publishing Company, İnc, 1975, pp. 301-357

L.A.Zadeh. Linguistic Approach and its Application to Decision Analysis // Directions in Large-Scale Systems, Plenum Press 1976, pp. 338-361.

Q.C.İmanov, N.F.Tağıyev, F.C.Məmmədov Müasir iqtisadiyyat, Neyron şəbəkələri, Fazi məntiq: Monoqrafiya. Bakı: Elm, 2003, 198 s.

К.Иманов, Р.Акперов . Нечеткие модели оценки качества социальных систем // LAPLAMBERT AcademicPublishing, Saarbrüken 2013, 60 p

İmanov G. J. et.all (2017). “Forecasting of oil and gas extraction in Azerbaijan on the basis of Hubbert’s model”, The jurnal of economic sciences: Theory and practice, pp.40-52, Volume 71, #1/2014, ISSN 2220-8739.

F.A.Cowell, E.Flachaire. Measuring mobility. Quantitative Economics 9(2018), 865-901

G.S.Fields and E.A.OK. The measure of income mobility: an introduction to the literature// Economic Research Report, 1996a

P.Van Kerm. What lies behind income mobility? Reranking and distributional change in Belgium, West Germany and the USA// Economica, 71(2004),223-239

M.Peng, D.Bu, Y.Wang. The Measure of Income Mobility in Vector Space. Physics Procedia 3 (2010) 1725-1732 pp.

G.S.Field and E.A.Ok, The meaning and measurement of income mobility,Journal of Ecomomic Theory,71(1996b) 347-377

G.S.Fields and E.A.OK, Measuring movement of income, Economica, 66, (1999) 455-471

G.S.Fields, Many facet in Economic Mobility, working paper, 2006

G.S.Fields, Income mobility, ILR collection, working paper, 2007.

T.Matra and E.A.OK, The measurement of income mobility: partial ordering approach,Economic Theory, 12(1998) 77-102.

V.Dardanoni, On measuring social mobility, Journal of Economic Theory.61(1993)372-394

M.D'Agostino and V.Dardanoni,The measurement on mobility: A Class of distance indices", working paper, (2006)

M.D'Agostino and V.Dardanoni, What's so special about Euclidean distance? A characterization with applications to mobility and spatial voting, Social Choice Welfare, 33(2009a) 211-233http://sticerd.lse.ac.uk/seminarpapers/special11042006.pdf

M.D'Agostino and V.Dardanoni, The measurement of rank mobility, Journal of Economic Theory, 144(2009b) 1783-1803.

D.Checchi and V.Dardanoni, Mobility comparisons: Does using different measures matter?, working paper(2002). http://www.economia.unimi.it/uploads/wp//wp46.pdf

D.Checchi and V.Dardanoni, Social mobility, in inequality and economic integration, Routledge, F.Farina and E.Savaglio editors, (2006) http://digitalcommons.ilr.cornell.edu/cgi/viewcontent.cgi article=&1018&context=workingpapers

N.Ding Y.Wang, A new approach to measuring the Economic mobility, working paper, 2006.

P. Van Kerm, Comparisons of income mobility profiles, IRISS working papers series, 2006. http://iriss.ceps.lu/documents/irisswp58.pdf

J.P.Rodriguez, J.G.Rodriguez, R.Salas. Income mobility and economic inequality from a regional perspective // Journal of Applied Economics, Volume 13, Issue 2, 2010, 335-350 pp. http://www.oecd.org/eco/growth/OECD-Note-EquivalenceScales.pdf

B. Buhmann, L.Rainwater, G.Schmauss and T.Smeeding (1988) 'Equivalence scales, well-being, inequality and poverty: sensitivity estimates across ten countries using the Luxembourg Income Study Database', The Review of Income and Wealth 34, 115-42.

H.Theil (1967) Economics and information theory, North Holland, Amsterdam.

U.Ebert (1997) 'Social Welfare when Needs Differ: An Axiomatic Approach', Economica, 64, 233-44.

www.azstat.gov.az

T.Bogomolova. Trajectories of Movement in the Space between Poverty and Non-Poverty of the Russian Population in the 1990s and 2000s // Russian Economic Journal Eco №5, 2011.–109-120 pp.

В.Н.Бобков (2009). Каким быть региональному неравенству качества и уровня жизни? // Мир России. Т. 18. № 3. С. 61–84.

M.Yunus. Can wealth concentration be stopped? The jurnal of economic sciences: Theory and practice, pp.40-52, Volume 73, #2/2016, ISSN 2220-8739

Hasanli Y. (2013). The evaluation of mutual substrution elasticity of capital and labour factors by application CES function for economy of Azerbaijan, The journal of economic
sciences: Theory and practice, p.77-96,2013, Year: 19, Volume 70, #1/2013, ISSN 2220- 8739

M.G. Thomason. Convergence of powers of a fuzzy matrix. Journal of Mathematical Analysis and Applications, 57:476{480}, 1977.

K.E.Avrachenkov, E. Sanchez. Fuzzy Markov Chains and decision-making // Fuzzy Optimization and Decision Making, 1(2):143{159}, 2002

R.Case. Fuzzy Markov chains and experiments with fuzzrank, an alternative to pagerank: A thesis for the degree of Master of Science. Queen's University Kingston, Ontario, Canada, 2016, 59 pp

B.F.Vajargah, M.Gharehdagi. Ergodicity of Fuzzy Markov Chains Based on Simulation Using Sequences. Journal of mathematics and computer Science 11(2014) 159 - 165

Chin Win Cheong, Amy Lim Hui, V. Ramachandaran, " Web Server Work Load Forecasting – Fuzzy Linguistic Approach", IJCIM, Volume 9, Number 3, September – December, 2001, 36-44 pp.